where the $T_c$ is the observed transition temperature. The values of $\lambda$ that correspond to each of our samples are given in Table I.

It appears, therefore, that the effect reported here may be due to the formation of very small grains in the aluminum alloy owing to URQ, and that the presence of a small quantity of Cu is only incidental to the physical picture. However, pure UR-quenched aluminum shows no such effect, at least not nearly as great. This is probably due to the fact that the addition of Cu impurity greatly facilitates the formation of small platelike grains which are difficult to obtain in pure aluminum no matter how great the quenching speed. This hypothesis is further strengthened by other observers,[17] who find in working with Cu-doped quenched Al films that these crystallize in smaller grains than when pure metal is used.

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# Motion of the Order Parameter in Type-II Transition-Element Superconductors in a High Magnetic Field

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The resistive state of type-II transition-element superconductors in a high magnetic field is studied using a technique due to Maki *et al.* The order parameters for the individual bands have similar Abrikosov-type solutions. Assuming a common upper critical field for the two bands, it is seen that the motions of the *s*- and *d*-electron pairs are controlled by different diffusion equations. The diffusion constants for the *s*- and *d*-pair motions are obtained for a typical transition-element superconductor. It is found that the *s*-band diffusion constant is not affected by the presence of the second band, but that the *d*-band diffusion constant is affected.

## I. INTRODUCTION

When placed in a perpendicular magnetic field, a type-II superconductor exhibits a triangular array of vortices through which the magnetic field penetrates the superconductor. Using the Ginsburg-Landau equations, Abrikosov[1] showed the existence of a mixed state containing an array of quantized flux lines. Recently, a number of experiments[2] have been performed to investigate the motion of the vortex lines caused by an externally applied electric field. Many theories,[3] based on the analogy between this situation and a similar one in superfluid helium, have been proposed to account for the dissipative phenomena encountered in the experiments. Most of these theories have dealt with the low-field region ($H \gtrsim H_{c1}$).

Schmid[4] has recently discussed the problem, using the time-dependent Ginzburg-Landau equations. It was seen that close to $H_{c2}$ and in the presence of a transverse electric field, the order parameter moves with an uniform velocity $u = E/H_{c2}$. Furthermore, he was able to account for the flux-flow resistivity close to the critical temperature. In a series of papers, Maki *et al.*[5] extended the approach to both pure and dirty type-II superconductors at arbitrary temperatures and in the vicinity of the upper critical field. It turns out that the calculations for dirty superconductors are much simpler than those for pure superconductors.

In all of Maki's papers, it has been assumed that the Bardeen-Cooper-Schrieffer (BCS) theory was adequate in describing the superconducting states. However, experimental measurements[6] of various properties of the transition-metal superconductors indicate that the BCS model is not adequate for these superconductors. A two-band model[7] has been introduced to account for the superconductivity in these elements. Using the two-band model, several authors[8] were able to obtain a best fit of the experimental data on the specific heats and the temperature variation of the penetration depths in superconducting niobium. More recently, direct evidence for a second transition temperature[9] and energy gap[10] in niobium has been obtained.

The aim of this paper is to apply the techniques developed by Maki *et al.* to transition-metal superconductors in high magnetic fields. Only the pure superconductors need to be considered, since it has been pointed out[11] that the dirty transition-element superconductors behave exactly like the BCS type. To ensure the presence of two energy gaps, we shall restrict ourselves to transition-metal superconductors having $g_{sd}^2 \ll g_s g_d$.

In Sec. II, we will expand Maki's formalism so that it can be applied to two-band superconductors. As we shall see, the physical observables in the two-band model are expressed as sums of two Maki-like observables. However, the energy-gap equations are quite different from Maki's expression. It is seen that by letting $g_{sd} = 0$, the energy-gap equations reduce to two Maki-like equations. (It should be pointed out that the energy gaps in the two-band superconductors are not the order parameters as is the case with pure one-band superconductors.)

The two energy gaps are obtained in Sec. III in terms of Abrikosov-like solutions. Both gaps will be of the following form:

$$\Delta^\dagger_{s(d)}(r,t) = \sum_n C^*_{s(d)n} \exp[ikn(y+ut)] \times \exp[-eH(x + kn/2eH + iu/4e\mathfrak{D}_{s(d)}H)^2] ,$$

where the diffusion constants $\mathfrak{D}_{s(d)}$ reflect the two-band nature of the model. In the absence of the electric fields, the two energy gaps are just the solutions[12] obtained by solving the two-band analog[13] of the Ginzburg-Landau equations. Assuming that $g_s \gg g_d$,[14] we find that the $s$-band energy gap is not affected much by the presence of the other band. However, it is seen that the $d$ band is affected by the $s$ band.

## II. FORMULATION

In this section, we will develop a general formulation with which various physical quantities in a state having a time-varying energy gap can be calculated. In studying the resistive state of type-II transition-metal superconductors in high magnetic fields, it becomes possible to treat the effects of the energy gap as a perturbation. (Near $H_{c2}$, the energy gaps or the order parameter in the case of a one-band superconductor will be extremely small.) We shall assume that there is only one upper critical field at which both energy gaps vanish.

The interaction Hamiltonian for the system is given by

$$\begin{aligned} H_I = {} & e\int n_s(r,t)\phi(r,t)d^3r + \int [\Delta_s(r,t)\Psi^\dagger_s(r,t) \\ & + \Delta^\dagger_s(r,t)\Psi_s(r,t)]d^3r \\ & + e\int n_d(r,t)\phi(r,t)d^3r + \int[\Delta_d(r,t)\Psi^\dagger_d(r,t) \\ & + \Delta^\dagger_d(r,t)\Psi_d(r,t)]d^3r , \end{aligned} \qquad (2.1)$$

where

$$\begin{aligned} & n_{s(d)}(r,t) = \sum_\sigma \psi^\dagger_{s(d)}\psi_{s(d)\sigma} , \\ & \Psi_{s(d)}(r,t) = \psi_{s(d)\uparrow}(r,t)\psi_{s(d)\downarrow}(r,t) , \\ & \Psi^\dagger_{s(d)}(r,t) = \psi^\dagger_{s(d)\downarrow}(r,t)\psi^\dagger_{s(d)\uparrow}(r,t) , \end{aligned} \qquad (2.2)$$

and

$$\begin{aligned} & \Delta^\dagger_s(r,t) = -g_s\langle\Psi^\dagger_s(r,t)\rangle - g_{sd}\langle\Psi^\dagger_d(r,t)\rangle , \\ & \Delta^\dagger_d(r,t) = -g_d\langle\Psi^\dagger_d(r,t)\rangle - g_{sd}\langle\Psi^\dagger_s(r,t)\rangle . \end{aligned} \qquad (2.3)$$

The operators $\psi^\dagger_{s(d)\sigma}$ and $\psi_{s(d)\sigma}$ are, respectively, the creation and destruction operators of a $s(d)$ electron with spin $\sigma$. The only assumption made in (2.1) and (2.3) is that the generalized time-dependent Hartree-Fock approximation is valid. $\phi(r,t) = -Ex$ is the scalar potential which describes a constant electric field in the $x$ direction. The interaction Hamiltonian can be separated into two parts, each being the interaction Hamiltonian for the individual bands.

In the Suhl-Matthias-Walker[7] (SMW) model, the physical quantities for the system are just the sums of the same quantities for the individual bands. For example, the electric current in the two-band system is

$$j(r,t)=j_s(r,t)+j_d(r,t)\ , \tag{2.4}$$

where

$$j_{s(d)}(r,t)=-(i/2m_{s(d)})\sum_\sigma\{[\nabla\psi^\dagger_{s(d)\sigma}(r,t)]\psi_{s(d)\sigma}(r,t)-\psi^\dagger_{s(d)\sigma}(r,t)[\nabla\psi_{s(d)\sigma}(r,t)]\}\ . \tag{2.5}$$

Similarly, the heat current for the system is just the sum of the heat currents $j^h_s(r,t)$ and $j^h_d(r,t)$, defined as

$$j^h_{s(d)}(r,t)=-(i/2m_{s(d)})\sum_\sigma\{\dot\psi^\dagger_{s(d)\sigma}(r,t)[\nabla\psi_{s(d)\sigma}(r,t)]+[\nabla\psi^\dagger_{s(d)\sigma}(r,t)]\dot\psi_{s(d)\sigma}(r,t)\}\ . \tag{2.6}$$

In general, the physical quantities will be bilinear in the electron creation and destruction operators of both bands.

The physical observables are the expectation values of the above type of quantities and are of the following form:

$$\tilde A(r,t)=\langle\exp[i\int_{-\infty}^t H_I(t')\,dt']A(r,t)\times\exp[-i\int_{-\infty}^t H_I(t'')\,dt'']\rangle \tag{2.7}$$

$$=\langle\exp[i\int_{-\infty}^t H_{sI}(t')\,dt']A_s(r,t)\times\exp[-i\int_{-\infty}^t H_{sI}(t'')dt'']\rangle+\langle\exp[i\int_{-\infty}^t H_{dI}(t')\,dt']A_d(r,t)\times\exp[-i\int_{-\infty}^t H_{dI}(t'')\,dt'']\rangle\ , \tag{2.8}$$

where $H_{sI}(r,t)$ and $H_{dI}(r,t)$ are the interaction Hamiltonians for the individual bands and are, respectively, the first two terms of (2.1) and the last two. In obtaining the separation of (2.8), use has been made of the noncontribution of terms linear in either $\Psi_{s(d)}$ or $\Psi^\dagger_{s(d)}$, since these terms would not conserve the number of particles. Carrying out the expansion and then keeping only the lowest-order terms, we get

$$\tilde A(r,t)=\tilde A_s(r,t)+\tilde A_d(r,t),$$

where

$$\tilde A_{s(d)}(r,t)=-ie\int_{-\infty}^t dt'\int d^3r\,\langle[A_{s(d)}(r,t),n_{s(d)}(r',t')]\rangle\,\phi(r',t')+\int_{-\infty}^t dt_1\int_{-\infty}^t dt_2\int d^3l\int d^3m\{\langle[[A_{s(d)}(r,t),\Psi_{s(d)}(l,t)],\Psi^\dagger_{s(d)}(m,t_2)]\rangle+\langle[[A_{s(d)}(r,t),\Psi^\dagger_{s(d)}(m,t_2)],\Psi_{s(d)}(l,t)]\rangle\}\Delta^\dagger_{s(d)}(l,t)\Delta_{s(d)}(m,t_2)\ . \tag{2.9}$$

The retarded products $i\langle[A(t),\ B(t')]\rangle\,\theta(t-t')$ can then be obtained by using the temperature Green's-function techniques.[15]

The energy gaps (2.3) take the form

$$\Delta^\dagger_{s(d)}(r,t)=-g_{s(d)}\langle\exp[i\int_{-\infty}^t H_{s(d)I}(t')\,dt']\Psi^\dagger_{s(d)}(r,t)\exp[-i\int_{-\infty}^t H_{s(d)I}(t'')\,dt'']\rangle-g_{sd}\langle\exp[i\int_{-\infty}^t H_{d(s)I}(t')\,dt']\Psi^\dagger_{d(s)}(r,t)\exp[-i\int_{-\infty}^t H_{d(s)I}(t'')\,dt'']\rangle\ . \tag{2.10}$$

Again, by carrying out the expansion and keeping only the lowest-order terms, we obtain

$$\Delta^\dagger_{s(d)}(r,t)=ig_{s(d)}\int_{-\infty}^t dt'\int d^3r'\,\langle[\Psi^\dagger_{s(d)}(r,t),\ \Psi_{s(d)}(r',t')]\rangle\,\Delta^\dagger_{s(d)}(r',t')+ig_{sd}\int_{-\infty}^t dt'\int d^3r'\,\langle[\Psi^\dagger_{d(s)}(r,t),\Psi_{d(s)}(r',t')]\rangle\,\Delta^\dagger_{d(s)}(r',t')\ , \tag{2.11}$$

where everything said in the development of $\tilde A_{s(d)}(r,t)$ also applies here.

If we were to consider the dirty limit, the mathematics involved in solving (2.11) would be greatly simplified. However, as we have previously said, the dirty two-band superconductors can be adequately described by the one-band model which has been fully treated by Maki *et al*. In Sec. III, we will obtain the Abrikosov solutions of the energy gaps of a clean type-II transition-element superconductor in a high magnetic field and subjected to a transverse electric field.

## III. ABRIKOSOV SOLUTIONS

We shall see that, to a good approximation, both of the energy gaps have Abrikosov-type solutions, i.e.,

$$\Delta^{\dagger}_{s(d)}(r,t)=\sum_n C^{*}_{s(d)n}\exp[ikn(y+ut)]\times\exp[-eH(x+kn/2eH+iu/4e\mathfrak{D}_{s(d)}H)^2], \quad (3.1)$$

where the diffusion constant $\mathfrak{D}_{s(d)}$ is chosen so that certain terms in a later equation will vanish. It will be seen that for transition-metal superconductors having $g_s \gg g_d$, the diffusion constant for the $s$ band is not altered by the presence of the second band, while the constant for the second band is altered by the presence of the $s$ band.

Following Abrikosov, Gor'kov, and Dzyaloshinski,[16] we incorporate the effects of the external fields by means of the phase factor to obtain

$$\Delta^{\dagger}_{s(d)}(r,t)=-g_{s(d)}\int_{-\infty}^{t}dt'\int d^3r'\exp[iS(r,t;r',t')]\langle[\Psi^{\dagger}_{s(d)}(r,t),\Psi_{s(d)}(r',t')]\rangle_{E=A=0}\Delta^{\dagger}_{s(d)}(r',t')$$
$$-ig_{sd}\int_{-\infty}^{t}dt'\int d^3r'\exp[iS(r,t;r',t')]\langle[\Psi^{\dagger}_{d(s)}(r,t),\Psi_{d(s)}(r,t)]\rangle_{E=A=0}\Delta^{\dagger}_{d(s)}(r',t'), \quad (3.2)$$

where

$$S(r,t;r',t')=2e\int_{t'}^{t}\phi(t'')\,dt''-2e\int_{r'}^{r}A(r'')\,dr''. \quad (3.3)$$

The retarded products

$$i\langle[\Psi^{\dagger}(r,t),\Psi(r,t)]\rangle_{A=E=0}\,\theta(t-t')$$

are the electron-electron or hole-hole propagators in the pure metal in the absence of the external fields.

These propagators are easily evaluated using the temperature Green's-function techniques.[16] Equation (3.2) can then be written as[17]

$$\Delta^{\dagger}_{s(d)}(r,t)=g_{s(d)}\int d^3r'K_{s(d)}(r',r)\,\Delta^{\dagger}_{s(d)}(r',t'=t-|x-x'|/|v_{s(d)x}|)$$
$$+g_{sd}\int d^3r'K_{d(s)}(r',r)\,\Delta^{\dagger}_{d(s)}(r',t'=t-|x-x'|/|v_{d(s)x}|), \quad (3.4)$$

where

$$K_{s(d)}(r,r')=\int\frac{d\Omega}{4\pi}\,\frac{1}{|v_{s(d)x}|}\exp\left(-ieE\frac{(x+x')\circ|x-x'|}{|v_{s(d)x}|}-ieH(x^2-x'^2)\tan\phi\right)$$
$$\times\int\frac{d^3q}{(2\pi)^3}\exp[iq(r-r')]\left\{\ln\frac{2\gamma\omega_D}{\pi T}-\frac{1}{2}\left[\psi\left(\frac{1}{2}-\frac{iqv_{s(d)}}{4\pi T}\right)+\psi\left(\frac{1}{2}+\frac{iqv_{s(d)}}{4\pi T}\right)\right]\right\} \quad (3.5a)$$

and

$$v_{s(d)x}=v_{s(d)j}=\sin\theta\sin\phi. \quad (3.5b)$$

By expanding the energy gaps $\Delta^{\dagger}_{s(d)}$ in the series

$$\Delta^{\dagger}_{s(d)}(r,t)=\sum_n a_{s(d)n}\,\phi_{s(d)un}(r,t), \quad (3.6)$$

where

$$\phi^{*}_{s(d)un}(r,t)=(\Pi^{\dagger}_{s(d)})^n\exp[ik(y+ut)]\times\exp[-eH(x+k/2eH+iu/4e\mathfrak{D}_{s(d)}H^2)] \quad (3.7)$$

and the eigenfunctions of the diffusion operator

$$\Pi_{s(d)}=\left(\frac{\partial}{\partial t}-2ie\phi\right)+\mathfrak{D}_{s(d)}(i^{-1}\vec{\nabla}+2e\vec{A})^2, \quad (3.8)$$

with

$$\Pi^{\dagger}_{s(d)}=\Pi_{s(d)x}+i\,\Pi_{s(d)y},$$

we can use (3.4) as the means to determine the expansion coefficients $a_{s(d)n}$. We will choose $\mathfrak{D}_{s(d)}$ so that only the first term in the expansion survives.

Substituting the expansion (3.6) into (3.4) and using the orthogonality of the eigenfunctions, we obtain the following algebraic equations:

$$\sum_n\left(\langle sun|sum\rangle\delta_{nm}-g_s\langle sun|K_s|sum\rangle\right)a_{sun}=g_{sd}\sum_{n'}\langle dun'|K_d|sum\rangle a_{dun'}, \quad (3.9)$$

$$\sum_n\left(\langle dun|dum\rangle\delta_{nm}-g_d\langle dun|K_d|dum\rangle\right)a_{dun}=g_{sd}\sum_{n'}\langle sun'|K_s|dum\rangle a_{sun'}, \quad (3.10)$$

where $\langle s(d)un|$ is the short-hand notation for $\phi^{*}_{s(d)un}$ and

$$\langle aun|K|bum\rangle=\int d^3r\int d^3r'\,\phi^{*}_{aun}(r',t')\times K(r',r')\phi_{bum}(r,t)\Big|_{t'=t-|x-x'|/|v_x|}. \quad (3.11)$$

By keeping only terms of first order in $u, g_s, g_d$, and $g_{sd}$, we obtain

$$a_{su1}=g_s\frac{\langle su0|K_s|su1\rangle}{\langle su1|su1\rangle}+g_{sd}\frac{\langle du0|K_d|su1\rangle}{\langle su1|su1\rangle}, \quad (3.12)$$

$$a_{du1} = g_d \frac{\langle du0|K_d|du1\rangle}{\langle du1|du1\rangle} + g_{sd} \frac{\langle su0|K_s|du1\rangle}{\langle du1|du1\rangle} , \quad (3.13)$$

where $a_{su0}$ and $a_{du0}$ were set to unity. It is interesting to note that in the limit $u \to 0$, where the $s$ and $d$ designation is spurious, the coefficients $a_{su1}$ and $a_{du1}$ both go to zero since $\langle 0n|K(r,r')|0m\rangle$ is diagonal. This implies that in the absence of the electric field, the vortex structures identified with the electrons belonging to the different bands are identical to each other.

The diffusion constants are obtained by solving the following equations:

$$\frac{\langle su0\,|K_s\,|su1\rangle}{\langle du0\,|K_{\bar{\alpha}}|\,su1\rangle} = -\frac{g_{sd}}{g_s} , \quad (3.14)$$

$$\frac{\langle du0\,|K_d\,|\,du1\rangle}{\langle su0\,|K_s\,|\,du1\rangle} = -\frac{g_{sd}}{g_d} . \quad (3.15)$$

In the case $g_{sd}=0$, the two diffusion constants will be identical except for having different Fermi velocities. This case is informative even though it has been pointed out that the assumption $g_{sd}=0$ will lead to discrepancies in the values of the energy gaps.[18] It shows that even though the vortices associated with the different bands move at the same speed, the dissipations of energy by the two moving vortices are different from each other.

For transition-metal superconductors such as niobium where $g_{sd}^2 \ll g_s g_d$ and $g_s \gg g_d$, we see that $g_{sd}/g_d$ is extremely small and could be considered to be zero. Therefore, we obtain

$$\mathfrak{D}_s = \frac{v_s^2}{8\pi T} \int_0^1 \frac{dz}{(1-z^2)^{1/2}} \int_0^\infty d\zeta\,\zeta^2 \times \exp(-\rho^2\zeta^2)\left[\sinh\left(\frac{\zeta}{(1-z^2)^{1/2}}\right)\right]^{-1}$$

$$\times\left[\int_0^1 \frac{dz}{(1-z^2)} \int_0^\infty d\zeta\,\zeta(1-\rho^2\zeta^2) \times \exp(-\rho^2\zeta^2)\left(\sinh\frac{\zeta}{(1-z^2)^{1/2}}\right)^{-1}\right]^{-1} , \quad (3.16)$$

where

$$\rho = \epsilon/2\pi T , \quad \epsilon = v_s(\tfrac{1}{2}eH_{c_2})^{1/2} .$$

The asymptotic forms of the diffusion constant $\mathfrak{D}_s$ are

$$\mathfrak{D}_s = (v_s^2/2\pi T)(0.284 - 0.734\rho^2), \quad T \lesssim T_{cd} \quad (3.17)$$

$$\mathfrak{D}_s = (v_s^2/2\pi T)\{1 - (12\rho^2)^{-1}[\ln(\pi^2\gamma\rho^2)^{-1} + 1 + 2\zeta'(2)/\zeta(2)] + (2/\pi^2\rho^2)\zeta(\tfrac{3}{2})[1-2^{-1/2}]\} , \quad T \ll T_{cd} . \quad (3.18)$$

The evaluation of (3.15) is more complicated since the relatively large value of $g_{sd}/g_d$ prevents us from setting the right-hand side of (3.15) equal to zero. Instead of actually evaluating the matrix element $\langle su0|K_s\,|dul\rangle$, we will assume that

$$\langle su0|\,K_s\,|\,du1\rangle = CE/H_{c_2} , \quad (3.19)$$

which is reasonable since we are only interested in terms of first order or less in $u$. The zeroth-order term does not exist since in the limit $u=0$ the matrix element must vanish. With this assumption, we have

$$T\int \frac{d\Omega}{v_d \sin\theta\,|\cos\phi|} \int_{-\infty}^{\infty} d\zeta \times \frac{\exp(-eH^2\zeta^2/2\cos^2\phi)}{\sinh(2\pi T\,|\zeta|\,/v_d \sin\theta\,|\cos\phi|)} \times\left(-\frac{\zeta^2}{2\mathfrak{D}_d} + \frac{|\zeta|(1-e^2H\zeta^2\tan^2\phi)}{v_d\sin\theta\,|\cos\phi|}\right) = -C\frac{g_{sd}}{g_d}\frac{E}{H_{c_2}} . \quad (3.20)$$

Solving for $\mathfrak{D}_d$, we get

$$\mathfrak{D}_d = \frac{v_d^2}{8\pi T}\left[\int_0^1 \frac{dz}{(1-z^2)^{1/2}} \int_0^\infty d\zeta\,\zeta^2 \exp(-\rho^2\zeta^2)\left(\sinh\frac{\zeta}{(1-z^2)^{1/2}}\right)^{-1} - \frac{g_{sd}}{g_d}c'\frac{E}{H_{c2}}\right]$$

$$\times\left[\int_0^1 \frac{dz}{(1-z^2)^{1/2}} \int_0^\infty d\zeta\,\zeta(1-\rho^2\zeta^2)\exp(-\rho^2\zeta^2)\left(\sinh\frac{\zeta}{(1-z^2)^{1/2}}\right)^{-1}\right]^{-1} , \quad (3.21)$$

where

$$\rho = \epsilon/2\pi T, \quad \epsilon = v_d(\tfrac{1}{2}eH_{c_2})^{1/2} , \quad C' = (8\pi T/v_d^2)C .$$

It therefore appears that within the approximations used, the $d$-band diffusion constant depends on the interband electron-phonon coupling $g_{sd}$. In the sense that the coupling $g_{sd}$ is dependent on the density of states in the $s$ band, we can say that the $d$-band diffusion constant is modified by the presence of the $s$ band and that the $s$-band diffusion constant is not affected by the $d$ band.

## IV. ADDITIONAL COMMENTS

One of the more crucial assumptions made is the common upper critical field for the two bands. Otherwise, much of the discussion in Secs. II and III would not be valid. To see this, let us assume that there exist separate critical fields for the two energy gaps and identify them as $H_{cs}$ and $H_{cd}$, the fields at which the $s$ and $d$ energy gaps go to zero. Close to $H_{cd}$, the expansion implicit in (2.10) would still be possible. However, the $g_{sd}$ term in the equation for $\Delta_d(r,t)$ would be zero since it depends on $\Delta_s(r,t)$ which is zero above $H_{cs}$. This would result in an expression for $D_d$ different from (3.21). It would be completely impossible to obtain anything close to $H_{cs}$. Near this field, the power expansion in $\Delta_d(r,t)$ would diverge. This would make impossible the derivations found in Sec. III.

To see how there is only one upper critical field for the two bands, we need only look at (2.3). We can interpret the energy gaps as resulting from pair correlation of electrons in both bands at the same time. Looking at the $s$-band energy gap, we can see that even at fields where the correlation of electrons in the $s$ band is zero, the energy gap $\Delta_s(r,t)$ would not necessarily vanish. The $d$-band correlation would still exist. This is reminiscent of the continuing existence of the energy gaps above $T_{cs}$ seen by Hafstrom *et al.* in the tunneling experiments on niobium.


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# Theory of an Isolated Vortex in a Pure Superconductor near $T = T_c$*

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It is demonstrated that the theory of Bardeen, Kümmel, Jacobs, and Tewordt for an isolated vortex in a pure type-II superconductor at arbitrary temperatures is in agreement with the theory of Neumann and Tewordt near $T = T_c$ if nonanalytic terms of odd order in $(1 - T/T_c)^{1/2}$ in the free energy are assumed to vanish. The leading nonanalytic term is examined by the use of perturbation theory to see if it vanishes, but no definite conclusion can be drawn. The approximations used in deriving these results should prove useful in the application of the method of Bardeen *et al.* to other problems involving pure inhomogeneous superconductors.

## I. INTRODUCTION

Bardeen *et al.*[1] (BKJT) have recently presented a theory for the properties of an isolated vortex in a pure type-II superconductor at arbitrary temperatures, thus giving a partial solution to one of the outstanding problems in type-II superconductivity.[2] Among other results, $H_{c1}$ was obtained as a function of $\kappa$ at 0 °K; an extension to higher temperatures is in progress.[3] The isolated vortex problem is also being treated by Eilenberger and Büttner who have recently published a preliminary report[4]; their theory is not limited to pure superconductors.

Prior to these efforts, the best theory for an isolated vortex was that of Neumann and Tewordt[5]